

## Calcul des déformations des ressorts hélicoïdaux

### Ressort cylindrique - Force et couple concentrés

#### Flexion et torsion - Exemple numérique

Fil rond en acier

$$d := 5 \cdot \text{mm} \quad S := \pi \cdot \frac{d^2}{4} \quad E := 2.0 \cdot 10^5 \cdot \text{N} \cdot \text{mm}^{-2} \quad G := \frac{E}{2.5} \quad \rho := 7.85 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$$

➔ Référence : E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$J_t := J_{t\_circ}(d) \quad I_{22} := I_{f\_circ}(d) \quad I_{33} := I_{22}$$

$$W_t := W_{t\_circ}(d) \quad W'_t := W_t \quad W_{f2} := W_{f\_circ}(d) \quad W_{f3} := W_{f2}$$

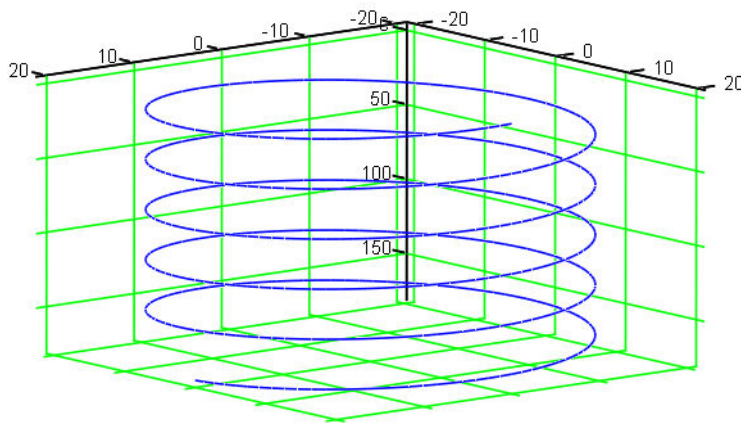
Caractéristiques du ressort  $D := 40 \cdot \text{mm} \quad R := 0.5 \cdot D \quad n_{sp} := 5.25 \quad \psi_{AB} := 2 \cdot \pi \cdot n_{sp}$

$$\beta := 15 \cdot \text{deg} \quad p := 2 \cdot \pi \cdot R \cdot \tan(\beta) \quad s(\alpha) := R \cdot \alpha \cdot \cos(\beta)^{-1} \quad L := s(\psi_{AB}) \quad L = 68.301 \text{ cm}$$

Forme du ressort

$$x_0(\alpha) := R \cdot \cos(\alpha) \quad y_0(\alpha) := R \cdot \sin(\alpha) \quad z_0(\alpha) := R \cdot \tan(\beta) \cdot \alpha$$

$$n := 100 \cdot n_{sp} + 1 \quad i := 0 \dots n - 1 \quad \alpha_{0_i} := \frac{\psi_{AB}}{n - 1} \cdot i \quad x_i := x_0(\alpha_{0_i}) \quad y_i := y_0(\alpha_{0_i}) \quad z_i := z_0(\alpha_{0_i})$$



$$\left( \frac{x}{\text{mm}}, \frac{y}{\text{mm}}, \frac{z}{\text{mm}} \right)$$

Forces extérieures excentrées à l'extrémité inférieure du ressort  $\psi_F := \psi_{AB} \quad \chi_F := \arctan\left(\frac{R}{2 \cdot |z_0(\psi_F)|}\right)$

$$P := 50 \cdot \text{N} \quad r_F := z_0(\psi_F) \cdot \cos(\chi_F)^{-1} \quad \lambda_F := 60 \cdot \text{deg} \quad \gamma_F := 45 \cdot \text{deg} \quad \chi_F = 3.238 \text{ deg}$$

$$\mathbf{F} := P \cdot (\cos(\lambda_F) \cdot \sin(\gamma_F) \quad \sin(\lambda_F) \cdot \sin(\gamma_F) \quad \cos(\gamma_F))^T \quad \mathbf{F}^T = (17.678 \quad 30.619 \quad 35.355) \text{ N}$$

$$C := 5 \cdot \text{N} \cdot \text{m} \quad r_C := z_0(\psi_F) \cdot \cos(\chi_F)^{-1} \quad \lambda_C := 150 \cdot \text{deg} \quad \gamma_C := 45 \cdot \text{deg}$$

$$\mathbf{C} := C \cdot (\cos(\lambda_C) \cdot \sin(\gamma_C) \quad \sin(\lambda_C) \cdot \sin(\gamma_C) \quad \cos(\gamma_C))^T \quad \mathbf{C}^T = (-3.062 \quad 1.768 \quad 3.536) \text{ mN}$$

➡ Référence :E:\Résonateur (TA)\Ressorts hélicoïdaux\Ressort hélicoïdal E\_L - F&C.mcd(R)

### Torseur des forces de cohésion

$$\alpha_M := 40 \cdot \text{deg} \quad \mathbf{R}_c(\alpha_M)^T = (17.678 \quad 30.619 \quad 35.355) \text{ N}$$

$$\mathbf{M}_c(\alpha_M)^T = (-8.461 \quad 5.368 \quad 3.117) \text{ N} \cdot \text{m}$$

### Sollicitations

Traction-compression  $N_c(\alpha_M) = 20.831 \text{ N}$

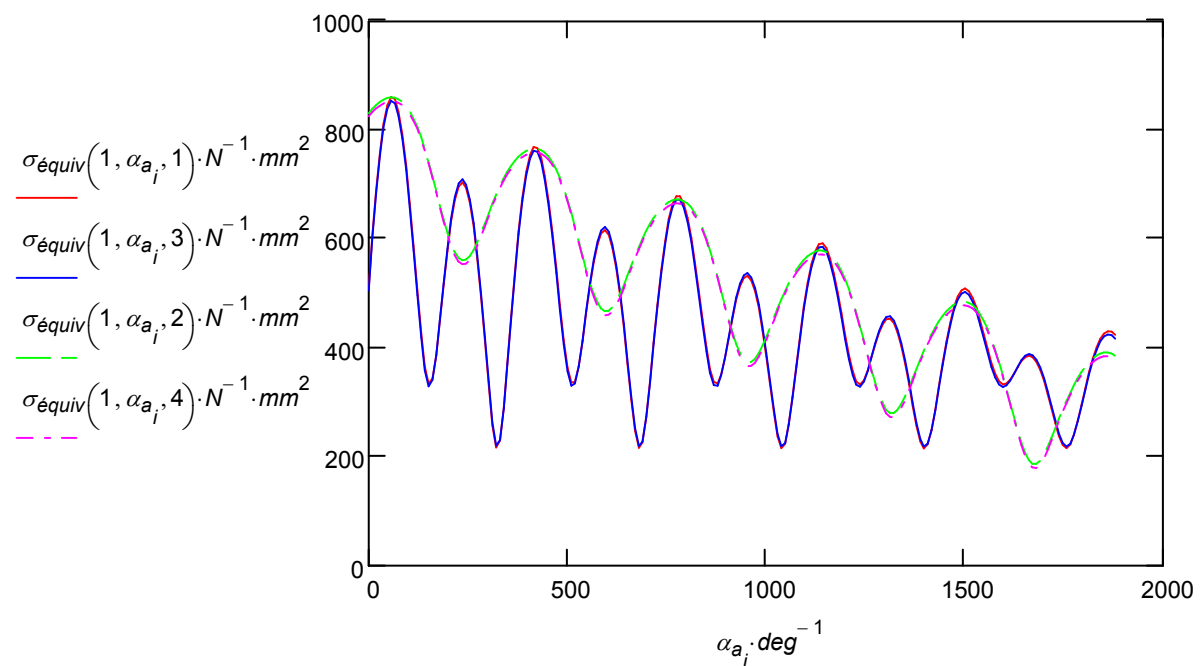
Efforts tranchants  $Q_2(\alpha_M) = -33.223 \text{ N} \quad Q_3(\alpha_M) = 31.021 \text{ N}$

Moment de torsion  $M_t(\alpha_M) = 10.032 \text{ N} \cdot \text{m}$

Moments de flexion  $M_{f2}(\alpha_M) = 3.031 \text{ N} \cdot \text{m} \quad M_{f3}(\alpha_M) = 0.539 \text{ N} \cdot \text{m}$

### Contraintes

$$n := 201 \quad i := 1 \dots n - 1 \quad \alpha_{a_i} := (i - 1) \cdot \frac{\psi_{AB}}{n - 1}$$



### Calcul des déplacements linéiques

#### Position du déplacement désiré

$$\alpha_M := \psi_{AB}$$

#### Déplacement dans la direction de Ox

$$\lambda := 0 \cdot \text{deg} \quad \gamma := 90 \cdot \text{deg}$$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = 23.299 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = 17.712 \text{ mm}$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = -5.31 \text{ mm}$$

$$\delta_x(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$\delta_x(\alpha_M) = 35.702 \text{ mm}$$

#### Déplacement dans la direction de Oy

$$\lambda := 90 \cdot \text{deg} \quad \gamma := 90 \cdot \text{deg}$$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = 37.453 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = 30.382 \text{ mm}$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = 2.656 \text{ mm}$$

$$\delta_y(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$\delta_y(\alpha_M) = 70.491 \text{ mm}$$

Déplacement dans la direction de Oz     $\lambda := 0 \cdot \text{deg}$      $\gamma := 0 \cdot \text{deg}$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = -2.46 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = -5.918 \text{ mm}$$

$$\delta_z(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = -1.98 \text{ mm}$$

$$\delta_z(\alpha_M) = -10.357 \text{ mm}$$

### Calcul des déplacements angulaires

Déplacement angulaire autour de Ox     $\lambda_c := 0 \cdot \text{deg}$      $\gamma_c := 90 \cdot \text{deg}$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = -21.2 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = -16.953 \text{ deg}$$

$$\theta_x(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = -0.757 \text{ deg}$$

$$\theta_x(\alpha_M) = -38.91 \text{ deg}$$

Déplacement angulaire autour de Oy     $\lambda_c := 90 \cdot \text{deg}$      $\gamma_c := 90 \cdot \text{deg}$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = 12.764 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = 10.092 \text{ deg}$$

$$\theta_y(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = 1.073 \text{ deg}$$

$$\theta_y(\alpha_M) = 23.929 \text{ deg}$$

Déplacement angulaire autour de Oz     $\lambda_c := 0 \cdot \text{deg}$      $\gamma_c := 0 \cdot \text{deg}$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = 3.792 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = 0 \text{ deg}$$

$$\theta_z(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = 18.338 \text{ deg}$$

$$\theta_z(\alpha_M) = 22.13 \text{ deg}$$

### Graphe de la déformation

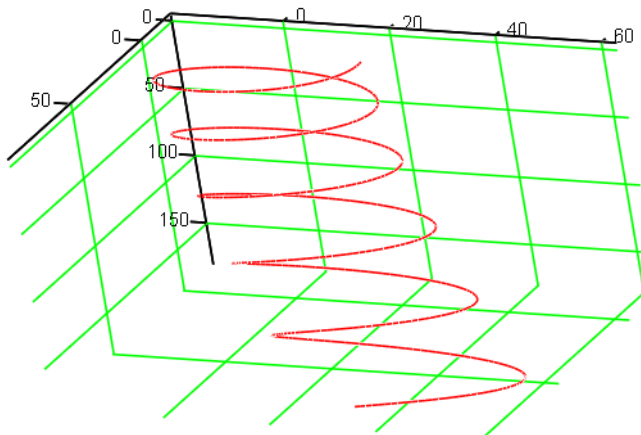
$$x_d(\alpha) := x_0(\alpha) + \delta_x(\alpha)$$

$$y_d(\alpha) := y_0(\alpha) + \delta_y(\alpha)$$

$$z_d(\alpha) := z_0(\alpha) + \delta_z(\alpha)$$

$$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha) \quad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha) \quad z'_d(\alpha) := \frac{d}{d\alpha} z_d(\alpha)$$

$$\mathbf{X} := \overrightarrow{x_d(\alpha_0)} \quad \mathbf{Y} := \overrightarrow{y_d(\alpha_0)} \quad \mathbf{Z} := \overrightarrow{z_d(\alpha_0)}$$



$$\left( \frac{X}{mm}, \frac{Y}{mm}, \frac{Z}{mm} \right)$$